Unconventional means of preventing chaos in the economy

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Abstract

This research explores five barriers, acting as obstacles to chaos in terms of policy (A), economic (B), social (C), demographic effects (D) and natural effects (E) factors for six countries: Russia, Japan, USA, Romania, Brazil and Australia and proposes a mathematical modelling using an original program with Matlab mathematical functions for such systems. The initial state of the systems is then presented, the element that generates the mathematical equation specific to chaos, the block connection diagram and the simulation elements. The research methodology focused on the use of research methods such as mathematical modelling, estimation, scientific abstraction. Starting from the qualitative assessment of the state of a system through its components and for different economic partners, the state of chaos is explained by means of a structure with variable components whose values are equivalent to the economic indices researched.

Keywords: chaos, system, Chiua simulator, multiscroll, barriers

Introduction

In the past four decades we have witnessed the birth, development and maturing of a new theory that has revolutionised our way of thinking about natural phenomena. Known as chaos theory, it quickly found applications in almost every area of technology and economics.

The emergence of the theory in the 1960s was favoured by at least two factors. First, the development of the computing power of electronic computers enabled (numerical) solutions to most of the equations that described the dynamic behaviour of certain physical systems of interest, equations that could not be solved using the analytical methods available at the time.

The second factor is the revolution in science triggered by quantum mechanics and the end of the era of Laplace's mechanistic determinism. Even though the birth of chaos theory is linked to the name of Henri Poincaré (1903), it was American meteorologist Edward Lorenz (1960) who would bring it to the attention of the scientific world. Chaos theory (the theory of complex systems) describes the behaviour of certain nonlinear dynamic systems, exhibiting the phenomenon of instability

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compared to the initial conditions, which is why their long-term behaviour is unpredictable, chaotic (Lorenz, 1984). Nonlinear behaviour allows a better understanding of complex natural phenomena. Nonlinear dynamics has introduced a set of new concepts and tools that allow the analysis and investigation of the dynamics generated by nonlinear processes. One can argue that currently there is a conceptual unification of notions (attractors, doubling of the period, bifurcations, Lyapunov exponent, sensitivity to the initial conditions). Techniques that study the concepts introduced by nonlinear dynamics are grouped under the generic label of nonlinear signal processing or nonlinear analysis (Anishchenko et al., 2007).

Chaos is a fundamental property of a dynamic system, which displays nonlinearity and sensitive dependence on the initial conditions. A small disturbance produces significant changes, while a large disturbance produces major changes - disaster. Several interconnected systems can only function if they are synchronised in the sense of graphically resulting in sharp peaks or smooth, according to the proposed purpose. The relationships established between systems feature complex links (loops) between the input and the output values, some areas exhibiting inverse links. The quantitative study of these systems is part of the chaos theory. Weather is one example of a chaotic system (storms, cyclones, hurricanes, typhoons, etc. develop over time). Weather represents the behaviour of all the molecules that make up the atmosphere. Similarly, the evolution of the human species and not only follows evolutionary leap points known as “eras”. In the real world, disturbances in society of a political, economic, social nature can occur (e.g. diseases, political unrest, family disputes). From the beating of a butterfly’s wings to the rhythms of the human heart, seemingly unrelated irregularities take on new meaning if one looks at them from the perspective of chaos theory. Outstanding scholars (David Ruelle, James Yorke, Michael Feigenbaum, Benoit Mandelbrot, Rene Thom, George Birkhoff, Andrey Kolmogorov, Aleksandr Lyapunov, Ilya Prigogine) have contributed to the establishment and development of a science that still has much to say in terms of understanding the complex world around us (Oestricher, 2007, p.8).

The aim of this paper is to highlight the state of complex chaos with impact values for political (A), economic (B), social (C), demographic (D) and natural (E) factors and its representation for six countries: Russia, Japan, USA, Romania, Brazil and Australia, using the standardised values [minimum 0, maximum 1] integrated in an original program with Matlab mathematical functions. The standardised values [minimum 0, maximum 1] were highlighted by 2 Chua simulators, which include fixed and variable (nonlinear) electro-technical elements enabling the representation in Matlab of the state of chaos and catastrophe. Within the global system, by using the six-stage resonance state, the chaos interactions for the six countries are highlighted.

1. Chua's chaos simulator

The Chua chaos simulator (Chua circuit) has multiple applications - in policy, economy, telecommunications, technical physics, nanomaterial synthesis, biology. It is of interest due to the multitude of adjustment parameters in the simulation processes. The Chua chaos simulator generally consists of two capacitors, an inductor, resistor and nonlinear resistor. It presents a variety of chaotic phenomena exhibited by more complex circuits, which makes it popular. It can be built easily at low cost using standard, reference electronic components. The process of investigating the dynamic modes of the Chua simulator consists of (Chua et al., 1993):

- determining and studying the possible constant movements corresponding to the ways of generating oscillations of different types;
- studying the mechanisms of occurrence of self-oscillations in the simulator;
- finding specific types of movements and researching their evolution the parameters of the studied system change;
- researching the bifurcations of movements, determining the values of parameter bifurcation;
- researching the mechanisms of triggering chaos in regular oscillations;
- building the elements of the bifurcation diagrams (parametric portraits) according to the parameters of the studied model.

The solution to each of the formulated problems is a separate stage in the process of modelling the nonlinear dynamics of the Chua simulator.

2. Literature review

Chaos is encountered in systems as a complex in electrical circuits, lasers, plasma physics, molecular and quantum physics, fluid flow, electrochemistry, in the process of chemical reactions as well as in systems such as the pendulum. In other words, chaos theories have recently found applications in almost all branches in the technical, medical, social, arts fields, etc. Elements of chaos theory can be applied to systems in order to be directed, controlled, manipulated. Lately, chaos theory has been successfully applied in economic, financial and social systems characterised by complexity. The analysis of these complex socio-economic processes helps above all to anticipate the uncertainty of chaos at certain stages and secondly to identify the factors that determine the state of chaos (Kenedy, 1993). The application of chaos in the field of social sciences is still in its early stages. From the *butterfly effect* discovered by Edward Lorenz to the *calculation of the universal constant* of

Mitchell Feigenbaum and the *concept of fractals* developed by Benoit Mandelbrot, who created a new geometry of nature, the science of chaos has become one of the most important scientific innovations of our time (Gleick, 2020). Edward Lorenz noted: "*A phenomenon that seems to occur randomly, in fact has an element of regularity that could be described mathematically. The flapping of a single butterfly's wing today produces a tiny change in the state of the atmosphere. Over a period of time, what the atmosphere actually does diverges from what it would have done. So, in a month's time, a tornado that would have devastated the Indonesian coast doesn't happen. Or maybe one that wasn't going to happen, does*." (Lorenz, 1972). Hense, in everyday life, we can see complexity in the flow of traffic, weather changes, population changes, organizational behaviour, changes in public opinion, urban development, etc. Complexity is also more common under the name of ***the edge of chaos*** (Alligood, 1997, pp.31-67).

Thus, the development of creative imagination, invention and innovation are of interest for individual and social transformations, which are inherent in creativity, in the state of wakefulness and, respectively, of creative dreaming, in self-organisation processes, in psychopathology, in artistic creation (chaotic music). It is appropriate to break down applied research into chaos into scientific and technical (engineering) applications. Work on engineering applications demonstrates the use of chaos and methods of controlling chaotic systems particularly for practical problems or at least demonstrates their feasibility. Scientific applications (in physics, chemistry, biology, economics), are vectorised in control theory and the use of methods to discover new properties and regularities in physical, chemical, biological behaviour in certain applications.

3. The initial state of systems

A chaotic system exhibits three simple defining characteristics (Cloyd, 2002; Dao, 2014):

- *Chaotic systems are deterministic* (they feature a determining equation that drives their behaviour).
- *Chaotic systems are sensitive to the initial conditions* (even a slight change in the starting point can lead to significantly different results).
- *Chaotic systems are neither random nor disordered*. Truly random systems are not chaotic, as chaos does involve order and patterns. Although it is a field of study in mathematics, chaos has applications in several other disciplines, including sociology and other social sciences. In social sciences, it involves the study of complex nonlinear systems (of social complexity). It does not imply disorder, but rather very complicated ordered systems. Nature, including certain cases of social behaviour, is extremely complex and the only

prediction we can make is unpredictability. Chaos theory concerns this unpredictability of nature and tries to make sense of it

A deterministic system is chaotic whenever its evolution depends significantly on initial conditions. The simulation model involves the construction of mathematical structures, which should represent, by analogy, the evolution of the investigated phenomena (in our case the induction of the chaos state) (Doncean, 2012, pp. 55-129). Based on the model of chaos induction and simulation through RLC components - resistance, inductance, capacitance - the determining factors in each barrier or obstacle are highlighted along with the implications on the general state of a system:

$$H= \sum H_i \, c_i^k$$

Where:

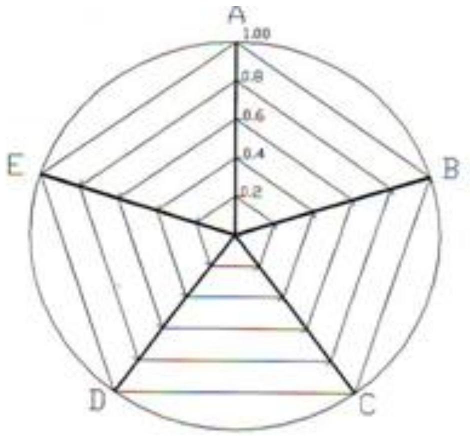
H – overall chaos

H_i – component chaos

c_i – importance coefficient

k – exponent

Figure 1. Geometric representation of chaos components with standardised minimum-maximum values in the [0; 1] interval



Source: Data processed by the authors

Assuming that for each country the impact of factors A, B, C, D, E is assessed based on qualitative estimated data. (Table no. 1, Figure no.1), then the state of components 1-5, labelled as j, is expressed by the following relation:

$$H_j=\sum \alpha_j \, C_j \tag{1}$$

Where H_j - extent of impact for component i (1…..5)

C_j - score of criterion j

α_j - importance coefficient

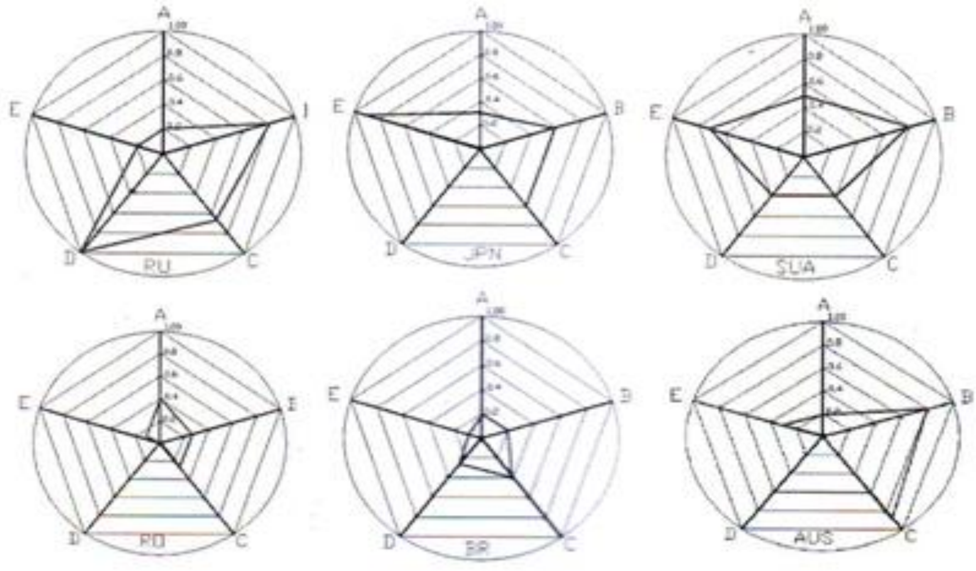
Table 1. Values of A, B, C, D, E impact factors with standardised values in the [minimum 0; maximum 1] range

No	Area	Values of impact factors				
		A. Political factor	B. Economic factor	C. Social factor	D. Demographic factor	E. Natural factors
1	**Russia (RU)**	0.2	0.8	0.5	1	0.2
2	**Japan (JPN)**	0.3	0.6	0.6	0	0.9
3	**USA**	0.5	0.8	0.4	0.4	0.5
4	**Romania (RO)**	0.4	0.3	0.3	0	0.1
5	**Brazil (BR)**	0.2	0.2	0.4	0.3	0.1
6	**Australia (AUS)**	0.2	0.8	0.9	0	0.3

Source: Qualitative data estimated by the authors

Initially, the factors with the values assessed qualitatively by the authors were estimated so that subsequently the state of chaos could be extended in the [0, 1] range by using 2 Chua chaos simulators with standardised values (see Annex 1).

Figure 2. Representation of chaos components by areas - countries (Russia, Japan, USA, Romania, Brazil, Australia) with the values of the standardised impact factors [0: 1]



Source: Data processed by authors (A - economic factor, B - economic factor, C - social factor, D - demographic factor, E - natural factor)

Table 2. Weight in overall chaos

No.	Chaos component	Proportion
1.	A. Political factor	**0.25**
2.	B. Economic factor	**0.25**
3.	C. Social factor	**0.20**
4.	D. Demographic factor	**0.15**
5.	E. Impact of natural factors	**0.15**
	TOTAL	**1**

Source: Data estimated by the authors

From the estimated data in Tables 1 and 2 respectively, following mathematical modelling and the Matlab user method we obtain the matrix consisting of five components of chaos: political (A), economic (B), social (C), demographic effects (D) and natural effects (E) for six systems: Russia, Japan, USA, Romania, Brazil, Australia, as follows:

$$A = \begin{bmatrix} 0.2000 & 0.3000 & 0.5000 & 0.4000 & 0.2000 & 0.2000 \\ 0.8000 & 0.6000 & 0.8000 & 0.3000 & 0.2000 & 0.8000 \\ 0.5000 & 0.6000 & 0.4000 & 0.3000 & 0.4000 & 0.9000 \\ 1.\,000 & 0 & 0.4000 & 0 & 0.3000 & 0 \\ 0.2000 & 0.9000 & 0.5000 & 0.1000 & 0.1000 & 0.3000 \end{bmatrix}$$

Where the columns include the estimated values for the political (A), economic (B), social (C), demographic (D), natural (E) factors, while the rows represent the six systems: Russia, Japan, USA, Romania, Brazil, Australia (related to table 3).

$A = A'$

$B = [0.2500\ 0.2500\ 0.2000\ 0.1500\ 0.1500]$, represents the weight of each factor in the overall total (pertaining to table 2).

Using the mathematical relation (1), and the relevant data in tables 1 and 2, we calculate for the relative chaos each country (table 3), and will obtain:

$$C=B*A \quad C=[0.5300\ \ 0.4800\ \ 0.5400\ \ 0.2500\ \ 0.2400\ \ 0.4750]$$

Table 3. Calculation of relative chaos

No.	Area	Chaos
1	**Russia**	0.2x0.25+0.8x0.25+0.5x0.2+1x0.15+0.2x0.15=**0.53**
2	**Japan**	0.3x0.25+0.6x0.25+0.6x0.2+0x0.15+0.9x0.15=**0.48**
3	**USA**	0.5x0.25+0.8x0.25+0.4x0.2+0.4x0.15+0.5x0.15=**0.54**
4	**Romania**	0.4x0.25+0.3x0.25+0.3x0.2+0x0.15+0.1x0.15=**0.250**
5	**Brazil**	0.2x0.25+0.2x0.25+0.4x0.2+0.3x0.15+0.1x0.15=**0.240**
6	**Australia**	0.2x0.25+0.8x0.25+0.9x0.2+0x0.15+0.3x0.15=**0.475**

Source: Data processed by the authors

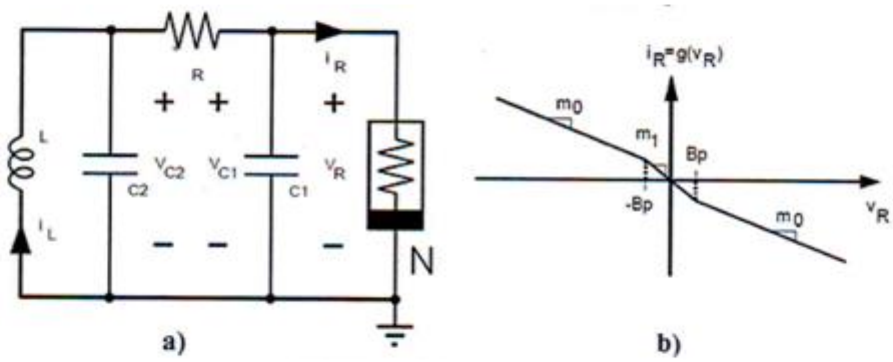
When analysing the above data, one can observe that for countries such as Russia, Japan, USA and Australia the values of general relative chaos H are in the range of 0.33 … 0.66, which corresponds to an average potential for chaos, tending to instability. For Romania and Brazil, their close values of general relative chaos H in the range 0 … 0.33 correspond to a small potential for chaos, considered closer to stability and balance. If the values of chaos were in the range of 0.66…1, then a high potential for chaos would result, with a high risk of catastrophe.

4. Mathematical modelling for the production and control of chaos

Based on the preliminary data in the first part of the paper, subsequent experiments (practical laboratory research) focus on the dynamic systems, the use of models based on chaos production and control, which could verify the data in previous tables (tables 1 and 2) .

The mathematical algorithm of chaotic control used presents certain distinct advantages in nonlinear dynamic economic simulation. (Figure no. 3). The key advantages are the constructive simplicity and the anticipation of the dynamic evolution of the state of chaos in the standardised interval [0,1].

Figure 3. Basic diagram of the Chua simulator



Source: Chua, (1992), The Genesis of Chua's Circuit pp.250-257

3. a) the Chua simulator (designed in 1983), where:

L - inductor,

R - linear resistance,

C_1, C_2- linear capacitors,

N - memistor (nonlinear resistance).

3. b) Representation of the v_i feature of the nonlinear resistor in the Chua simulator with m_0 slopes in the outer region and with m_1 slope in the inner region (+, -)

B_p - points of change in slope.

The Chua simulator in figure 3 a. is a simple oscillator that generates chaos and bifurcation. The simulator contains three linear energy storage elements (one inductor and two capacitors), a linear resistor and a single nonlinear resistor. Diodes, transistors, operating circuits, amplifiers can be used to make the nonlinear resistor.

The mathematical system for the Chua simulator consists of three ordinary differential equations, namely:

$$C_1 \frac{dv_{C_1}}{dt} = G(v_{C_2} - v_{C_1}) - g(v_{C_1})$$

$$C_2 \frac{dv_{C_1}}{dt} = G(v_{C_1} - v_{C_2}) + i_L \qquad L\frac{di_L}{dt} = -v_{C_2}$$

$$g(v_R) = m_0 v_R + \frac{1}{2}(m_1 - m_0)[| v_R + B_p | - | v_R - B_p |]$$

Where G = 1/R represents the equivalent conductance and $g(v_R)$ - linear representation.

According to Leon O. Chua, the following values are used for lab experiments (Chua et al., 1993):

L=18mH,

C_1=10nF,

C_2=100nF,

m_1=-(50/60)mS,

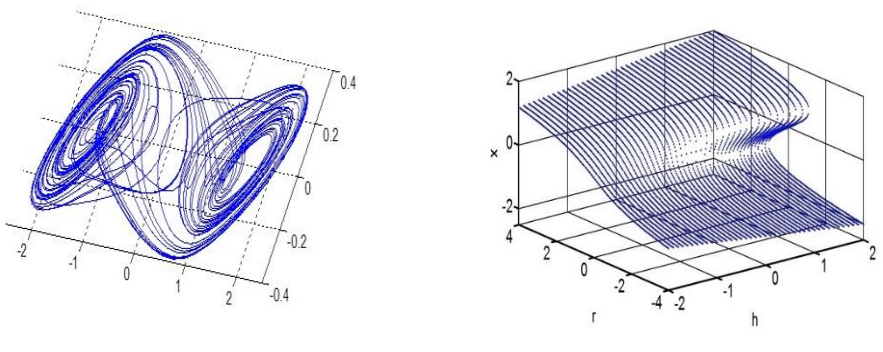
m_0=-(9/22)mS,

E=1V,

R=0….2kΩ .

For R = 1.73kΩ the x-y dependence is obtained which mimics the state of chaos (Chua, 1992). The following syntax are used in the Matlab processing environment: **chua_oscillator.m** and **run_lyap.m**.

Figure 4. Representation in Matlab of the state of chaos and the state of catastrophe



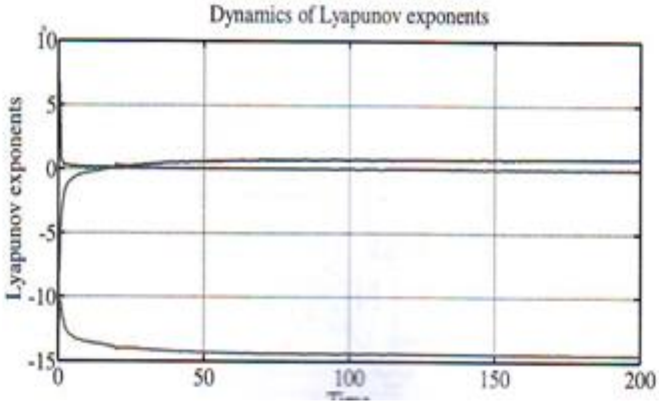
a. chaos ***b. catastrophe***

Sursa: Doncean Ghe. (2015), The acoustic characterisation of Chua's chaos circuit, p.233

In order to simulate the states of chaos and catastrophe, respectively, one should use ***relatively low-cost*** electro-technical circuits (chaos simulators), which would render for a macro-economic system the state of chaos or catastrophe (e.g., for political factors A, economic B, social C, demographic D and natural E and representation for six countries: Russia, Japan, USA, Romania, Brazil and Australia). This prevents disasters, which involve high costs and financial efforts.

In chaos theory, Lyapunov coefficients (exponents) are the most commonly used. *They represent asymptotic values that characterise the divergence or convergence of the neighbouring paths in the areas of dynamic system phases*. In other words, Lyapunov exponents refer to how two initially adjacent paths, separated by an infinitesimal interval, separate over time (Figure 5).

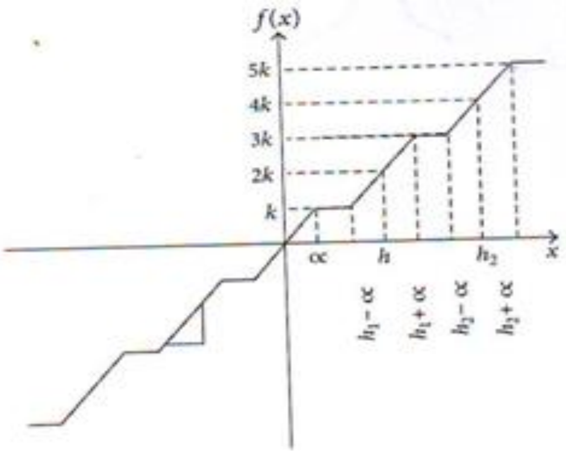
Figure 5. Determining the state of chaos using Lyapunov exponents



Source: Alligood, Kathleen, et al., (1997), Chaos: An Introduction to Dynamical Systems, p.49

This path exposes within a short period of time the expansion points of chaos. If one of the factors used exhibits impact disturbances, a chaos state develops. Technical solutions using several Chua simulators can be synchronised to obtain an image of a dynamic complex. Figure 6 shows the characteristics for five resistance levels (corresponding to the impact factors studied).

Figure 6. Graphical representation of the resonant state for 5 levels



Source: Data processed by the authors

The figure shows that 5 transition stages with interferences (corresponding to impact factors) and 6 stationary stages of a constant level (corresponding to the 6 countries).

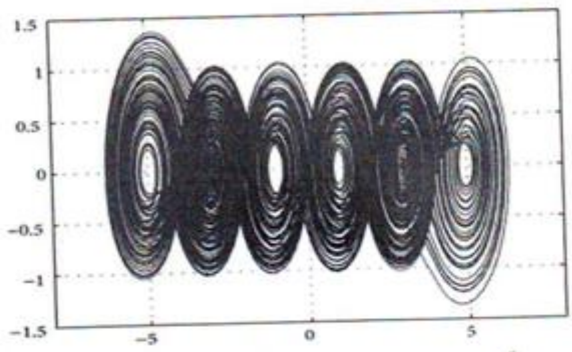
The related mathematical relations on these states are:

Stationary state	$(2q+1)k,\ x > qh + \alpha$
Growth state	$s(x\text{-}h)+2k,\ h\text{-}\ \alpha \leq\ x \leq h+a$
Stationary state	$s(x\text{-}2h)+4k,\ qh\text{-}\ \alpha \leq\ x \leq qh+\ \alpha$
Growth state	$sx,\ \text{-}\ \alpha \leq x \leq \alpha$
Stationary state	$s(x+2h)\text{-}4k,\ \text{-}qh\text{-}\ \alpha \leq x \leq \text{-}\ qh+\ \alpha$
Growth state	$s(x+h)\ \text{-}2k,\ \text{-}h\text{-}\ \alpha \leq x \leq \text{-}h+\ \alpha$
Stationary state	$\text{-}3k,\ \text{-}qh+\ \alpha < x < \text{-}\ h\text{-}\ \alpha$
Growth state	$\text{-}k,\ h\text{-}a < x < \text{-}\ \alpha$
Stationary state	$k,\ \alpha < x < h\text{-}\ \alpha$
Growth state	$3k,\ h+\ \alpha < x < qh\ \text{-}\ \alpha$
Stationary state	$\text{-}2+1k,\ x< \text{-}ph\text{-}\ \alpha$

where f(x; α, k, h, p, q) represent regulation variables that can be studied on this system.

Based on Figure 2, we argue that at the macroeconomic level connections emerge between the six countries, so that the simulation can present interlinks at the global level, establishing five transition stages with interferences presented in Figure 7.

Figure 7. Resonance study for five stages of transition with interferences

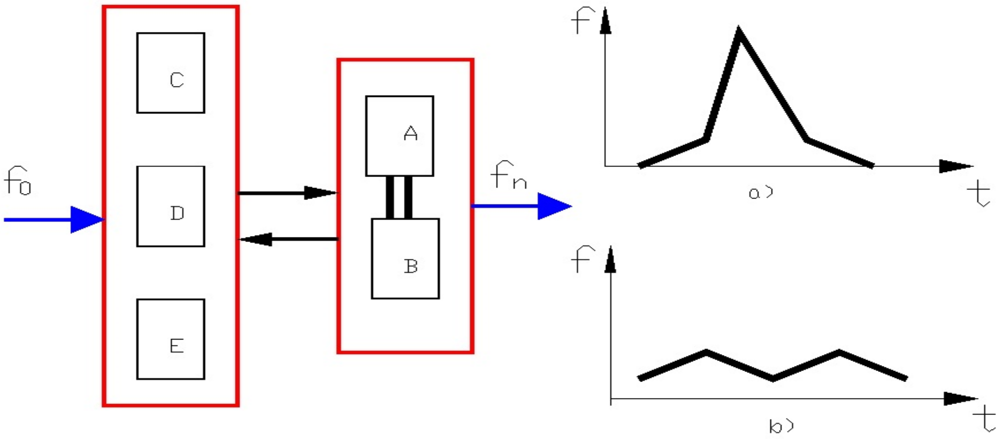


Source: Data processed by the authors

5. Block diagram of a simulation system

For the five barriers or obstacles to chaos, presented in the first part of the study, i.e. political (A), economic (B), social (C), demographic effect (D) and natural effects (E), Chua modules can help to create various connection schemes. Next, the Lyapunov coefficients (exponents) are calculated for each scheme. Finally, to indicate the state of chaos, the circuit itself is built and the simulation is then performed. For a given combination, the destruction of the system (Figure 8 above) or the linearisation with small disturbances (Figure 8 below) can be achieved.

Figure 8. Block diagram according to Lyapunov importance coefficients



Source: Data processed by the authors

In the graph one may note an F_0 - impact factor which when acting on a system determines a whole series of interconnections (feedback). The result F_n leads to solutions a (destruction of the system) or b (linearization with small disturbances).

Conclusions

The present paper deals with notions of the theory of nonlinear dynamics of chaotic and stochastic systems. It provides both an exhaustive introduction to the topic and a detailed discussion of the fundamental problems and the experimental results of the research. The aim of the paper is to highlight the state of complex chaos corresponding to the impact factors for six countries: Russia, Japan, USA, Romania, Brazil and Australia, using the standardised values [minimum 0, maximum 1] integrated in an original program with Matlab mathematical functions. The standardised values [minimum 0, maximum 1] were highlighted using two Chua simulators (entailing low costs), which include fixed and variable (nonlinear) electro-technical elements, allowing the representation in Matlab of the states of chaos and catastrophe. This prevents disasters that involve very high costs and financial efforts.

Chaos can be understood using basic knowledge of linear algebra and differential equations. The Chua chaos simulators presented are simple and provide a rich variety of phenomena: equilibrium points, periodic orbits, bifurcations and chaos. Such simulators have multiple applications - in politics, economics, telecommunications, technical physics, nanomaterial synthesis, biology - and are of interest thanks to the many adjustment parameters in the simulation processes.

In conclusion, starting from the qualitative assessment of the state of a system through its components: policy (A), economic (B), social (C), demographic effects (D) and natural effects (E) and for different economic partners*:* Japan, USA, Romania, Brazil and Australia and proposes a mathematical modelling, the state of chaos may be explained through a structure with variable components whose values are equivalent to the explored economic indices. Chua modules can help to create various connection schemes. Finally, to indicate the state of chaos, the circuit itself is built and the simulation is then performed.

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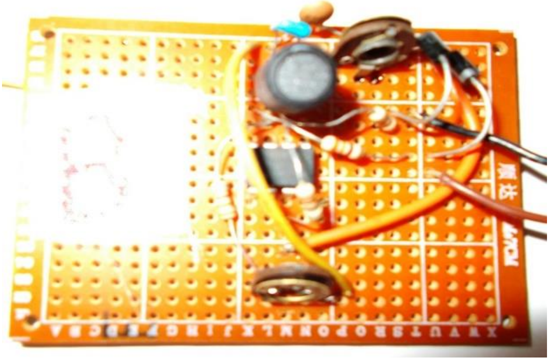
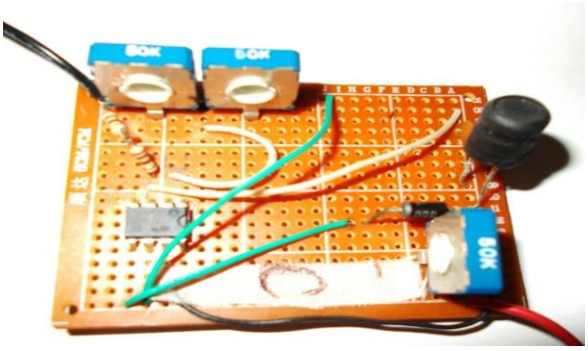
Appendix 1. Experimental laboratory models

Each area features specific oscillations of environmental resonance. This experiment determines the specific features of the oscillation. The goal is the maximum wave of intervention to sustain a certain type of reaction to resonance with minimum cost and energy.

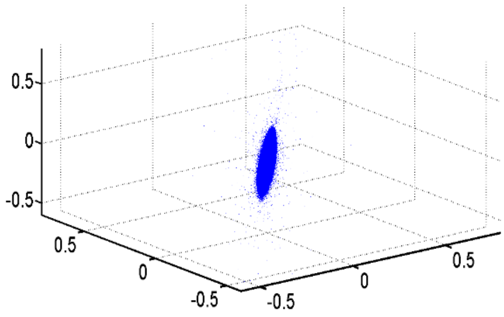
Chaos simulator A (electro-technical circuit)



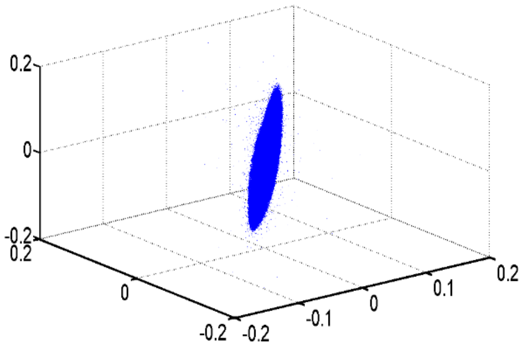
Chaos simulator B (electro-technical circuit)



3D graphic renders the spatial scattering and the emergence of chaos

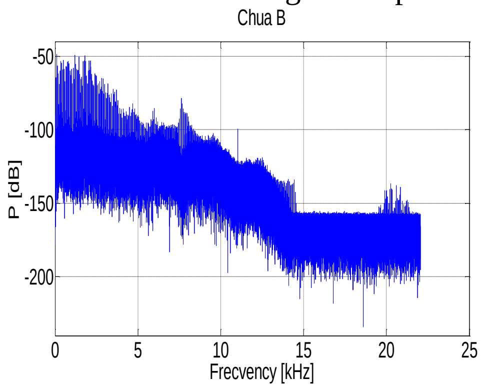


Chaos state for simulator A

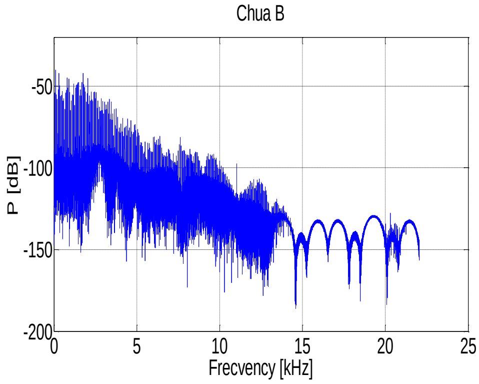


Chaos state for simulator A

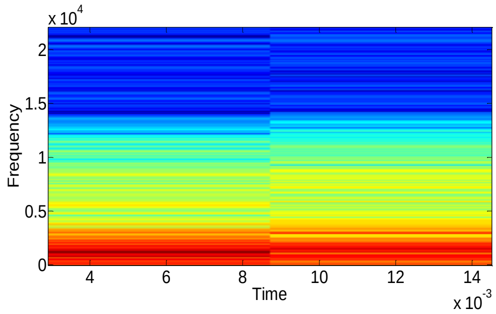
The change in amplitude and acoustic signal was highlighted.



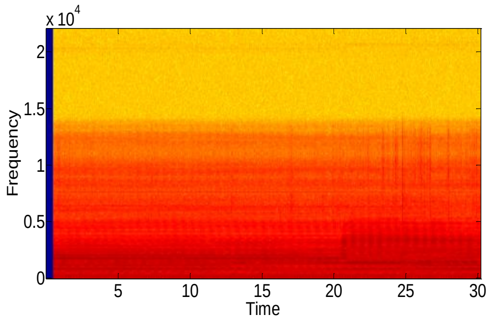
Specific features for simulator A



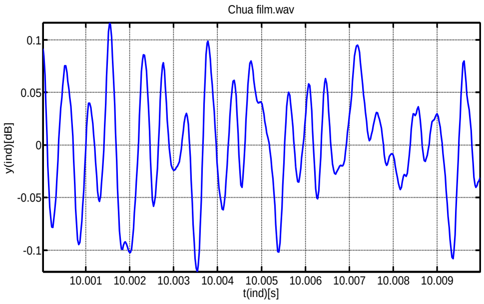
Specific features for simulator B



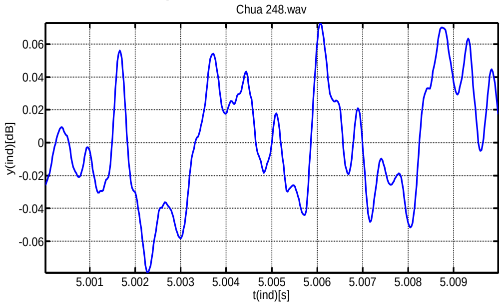
Sonogram for simulator A



Sonogram for simulator B



Different oscillations for simulator A



Different oscillations for simulator B

Source: Experimental data processed by the authors.